# A NONLINEAR INTEGER PROGRAMMING MODEL FOR EXPANDING THE TRANSPORTATION SYSTEM OF AN UNDERDEVELOPED COUNTRY OR REGION

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# THESIS

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## ABSTRACT

A nonlinear integer programming model for expanding the transportation system of an underdeveloped country is presented. The model uses integer 0-1 decision variables. The basic model has linear constraints and a nonlinear objective function. Some special situations and extensions to the model are presented. The benefits being maximized in the objective function are discussed, as are the problems of parameterization and suboptimization. A solution procedure for the model is suggested, but an efficient algorithm is not available for solving the model. Some areas for future research are also suggested.

TABLE OF CONTENTS

## LIST OF FIGURES

## TABLE OF SYMBOLS

| SYMBOL | DEFINITION |
|---|---|
| $A$ | Number of solutions to the model |
| $B_t$ | Budget for expenditure during time period $t = 1,2,...,T$ |
| $C_{ijkt}$ | Cost of constructing link $(i,j)$ of type $k = 1,2,...,K$ in time period $t = 1,2,...,T$ |
| $D$ | Number of decision variables in the model |
| $i$ | Discount rate |
| $(i,j)$ | Link in the transportation network; if $i \neq j$ then it is an arc between node $i$ and node $j$, and if $i = j$ then it is a node |
| $K$ | Number of link types in the transportation network |
| $L$ | Set of all technologically feasible links in the transportation network during the period of $T$ years |
| $L_p$ | Set of links $(i,j)$ that make up path $p = 1,2,...,P$, where $(i,j)\varepsilon L$ |
| $M_{ijkt}$ | Cost of maintaining/operating link $(i,j)$ of type $k = 1,2,...,K$ during period $t = 1,2,...,T$ |
| $N$ | Number of nodes in the transportation network |
| $n$ | Number of links in the transportation network; number of elements in the set $L$ |
| $P$ | Number of feasible paths from origins to destinations |
| $R_1$ | Set of $k$ such that $k$ is a road link |
| $R_2$ | Set of $k$ such that $k$ is a rail link |
| $R_3$ | Set of $k$ such that $k$ is a waterway link |
| $R_4$ | Set of $k$ such that $k$ is an airway link |
| $r_m$ | Number of elements in set $R_m$, $m = 1,2,3,4$ |
| $Q$ | Number of scarce material resources in the material constraints |
| $S$ | Set of all nodes that are origins |

| SYMBOL | DEFINITION |
|---|---|
| $\bar{S}$ | Set of all nodes that are destinations |
| $T$ | Number of years for which the transportation system is being planned |
| $V_{it}$ | Output from origin i during period $t = 1,2,\ldots,T$ |
| $v_{ijk}$ | Flow capacity of link $(i,j)\varepsilon L$ of type $k = 1,2,\ldots,K$ |
| $W$ | Total benefits derived from the transportation system during the entire planning period; value of the objective function |
| $W_{pkt}$ | Present value of benefits derived from the existence of path $p = 1,2,\ldots,P$ of type $k = 1,2,\ldots,K$ during period $t = 1,2,\ldots,T$ |
| $W^*_{pkt}$ | True value of $W_{pkt}$ (before discounting) |
| $X_{ijkt}$ | Indicator variable to show the existence of link $(i,j)$ of type $k = 1,2,\ldots,K$ in period $t = 1,2,\ldots,T$ |
| $x_{ijkt}$ | Decision variable to determine whether or not link $(i,j)$ of type $k = 1,2,\ldots,K$ is constructed during period $t = 1,2,\ldots,T$ |
| $x_{ijk0}$ | Variable used to express the existence of link $(i,j)$ of type $k = 1,2,\ldots,K$ prior to the beginning of the planning period |
| $Y^q_t$ | Quantity of resource q that is available for use in period t from national sources of supply |
| $y^q_{ijkt}$ | Quantity of resource q that is required to construct link $(i,j)$ of type k during period t |

## ACKNOWLEDGEMENT

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# I. INTRODUCTION

The U. S. Agency for International Development has pointed out that the economic growth of a nation is very closely related to the rate and type of expansion of its transportation system. [17] Countries that desire rapid economic growth often allocate a large portion of their annual budgets to the development of their transportation systems. Thus, it is imperative that this allocation be expended in an optimal manner. It is the purpose of this paper to formulate a mathematical resource allocation model that will optimally expand the transportation network of an underdeveloped country within the operating constraint of an annual budget.

First, it will be necessary to explain what is meant by the term underdeveloped country or region. The literature is not very precise or exact in defining an underdeveloped country. One way of defining an underdeveloped country is to say that it is a country whose inhabitants have a standard of living that is inferior to that enjoyed by people in developed countries. This definition is arbitrary and merely implies some relative status. However, one can go further and state that an underdeveloped country is one that has certain characteristic and basic problems. First, the prevailing per-capita level of income is low, compared to the developed countries. This may indicate that the per-capita gross national product is low or that the wealth and income of the nation is concentrated in the hands of a very small minority. Second, the inhabitants enjoy an inferior standard of living, compared to the developed countries. This includes such things as the amounts and types of food intake, infant mortality rates, life expectancies, literacy rates, and the lack of such basic facilities as roads, schools, telephones, electric power for the consumer, and adequate medical facilities, to cite merely a few. [11]

When a country is called underdeveloped, one means that it is economically underdeveloped, having low per-capita income that increases slightly, if at all. Such countries are generally not industrialized, and the majority of the population lives by low-yield agriculture. If the country does export a product, it is usually some type of raw material. It is generally agreed that these underdeveloped countries include most of Africa and Asia, the whole of Latin America, and a few countries in Europe. It must be emphasized that this definition refers to economic development. The developed countries include the United States, the Soviet Union, Great Britain, West Germany, France, and Canada. Since the definition is economic in nature, the underdeveloped countries may be well developed in other areas, such as art, music, religion, and literature. [8]

It is convenient, for the purposes of discussion and analysis, to divide the underdeveloped countries into two groups: countries with subsistence economies and raw-material countries. In a country with a subsistence economy, the first objective is normally to convert the nation from non-surplus farming to a market economy. Here the need arises for an improved transportation system and for market facilities. It is by means of an improved transportation system that the farmers reach the market, where the population is exposed to new ideas for improvements in production. Countries with subsistence economies are Nepal, Bhutan, Afghanistan, and some territories in Africa and in the Pacific. The raw-material countries have export production that is highly specialized and normally consists of one or two products in the nature of raw materials. This exploitation of natural resources usually leads to direct investment and to the importing of capital from abroad. As this occurs, it is important that the available capital is effectively and economically employed. Examples of raw-material countries are the oil producers (Iran, Iraq, Kuwait, and Venezuela), rubber producers (Malaysia and Vietnam), sugar producers (Cuba and the Dominican Republic), Tin exporter (Bolivia), copper exporters (Chile and Rhodesia), and the rice producers (Burma and Thailand). [8]

In many countries of the world, the mobility and accessibility that result from a transportation system are important to economic development. In the underdeveloped countries, immobility inhibits development, whereas in the developed countries mobility is essential to continued growth and prosperity. Fortunately for the underdeveloped nations, it is no longer necessary for a country to slowly evolve its transportation system through various stages, for they can reap the benefits of the technology that has been developed by the affluent nations. It must be realized that this is merely an alternative, for a specific underdeveloped country may not have the resources necessary to undertake an accelerated program of development or may not desire to rapidly develop its transportation system. As transportation facilities are improved, the economy tends to reflect this with industrial developments, taping of natural resources, and an increase in commercial activity. This indicates that the decisions with respect to the needs of the transportation system are not independent of other sectors in the economy. [17]

Various approaches have been suggested for solving the problem of how to expand the transportation system of an underdeveloped nation. Since the transportation system is not independent of other sectors in the economy, most analyses have taken the approach, whereby the key sectors in the economy are included in a model, so that interactions between sectors are implicitly or explicitly considered. Examples of this type of analysis are the input-output models, linear programming models that include several economic sectors, and matrix inversion models. [3,4,6,18]

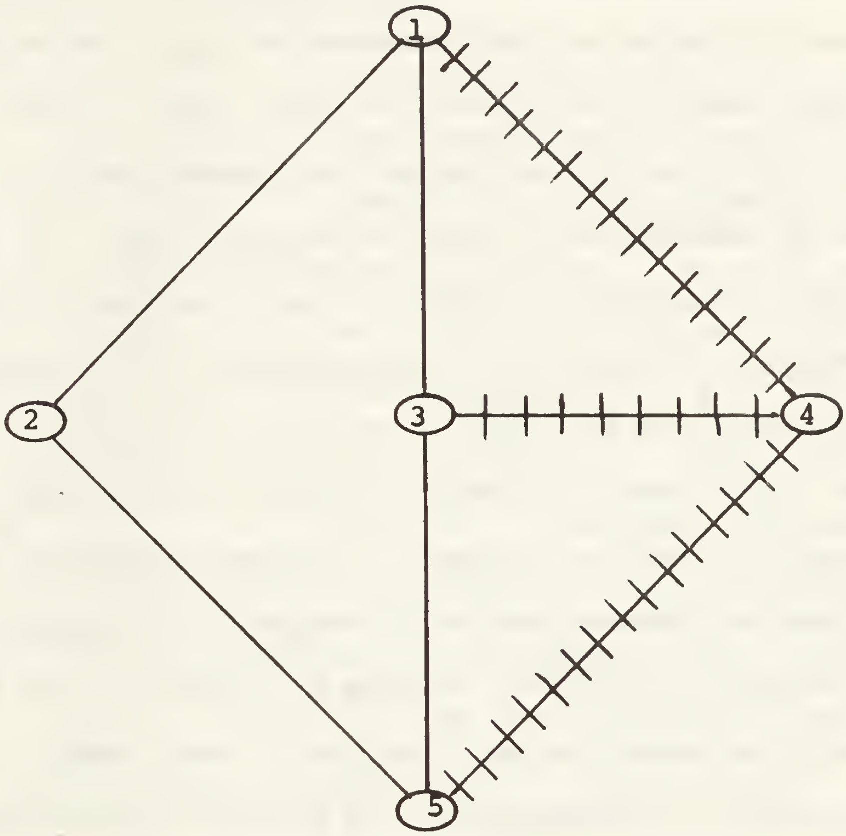
There are some instances when these approaches are not practical. An example is a nation in which the Ministry of Transportation is given a fixed budget and directed to improve the transportation system within the constraint of the budget. When one of the previously mentioned approaches is not practical, then a less desirable but acceptable method is to consider the alternative developments that can be made in the transportation system and to employ a model that selects the set of projects that will optimize the benefits derived by the underdeveloped country from the transportation system, while still operating under the constraint of a budget. The implicit assumption is that the transportation system is independent of other sectors. This assumption will be discussed more fully in Section IV, when benefits and the problems of suboptimization are discussed.

It is with these thoughts in mind that a model will be presented in Section II to optimize the benefits derived from the transportation system of an underdeveloped country. The model that will be presented can be used for short-range planning for periods of time of five to ten years to establish a transportation system to satisfy immediate needs or for long-range planning for periods of time of ten to twenty-five years to establish the scope of future construction. Estimates for periods less than five years are too short for effective practical use, since about three years are generally needed for financing, design, and construction. [17] The basic model that is presented in Section II is a very general model that may be used to plan the expansion of the transportation system in many underdeveloped countries. Due to this generality in the basic model, when it is applied for use in a particular nation, it must be recognized that certain modifications during the formulation stage may be required. To assist in accomplishing this, Section III discusses some special constraints and extensions of the basic model. Section IV discusses the objective function and the benefits that it optimizes, the exogenous parameters that are needed to use the model, a procedure for solving the model, the problem of suboptimization, and some conclusions. Finally, in Section V some areas for future research are suggested.

# II. THE BASIC MODEL

## A. DEFINITIONS AND NOTATION

In general, the network of a transportation system consists of a number of roads, highways, railroads, waterways, and air routes with their associated junctions, terminals, and intersections. Mathematically, the discussion is facilitated if the transportation system is treated as a network of nodes and arcs, which may also be referred to as facilities and routes, respectively. For example, consider the simple road and railroad network in Figure 1. The nodes are represented by the five circled numbers and could represent towns, intersections of roads, or transportation terminals. These are the facilities. The arcs are the portions of the network connecting two distinct nodes. The arc connecting nodes 1 and 2 is a road, whereas the arc connecting nodes 1 and 4 is a railroad. These arcs are the routes. The term link is used when one desires to refer to an arc or a node but does not desire to distinguish between an arc and a node. In order to refer to a specific link in the transportation network, the convention (i,j) will be used, where (i,j) refers to a node if $i = j$ and (i,j) refers to an arc if $i \neq j$. Thus, (1,4) refers to the rail route between nodes 1 and 4 and is an arc, and (2,2) refers to node 2 in Figure 1. The reason for this redundancy in specifying nodes will become clear later. The network is undirected, which means that traffic can flow in either direction on any arc. In addition, when referring to an arc (i,j), the notation is not to be taken as an ordered pair. Thus, (i,j) is the same as (j,k) in this paper.




Legend:

m —— n : Road between nodes m and n; $m \neq n$; $m,n = 1,2,3,4,5$

k ┼┼┼┼┼┼ j : Railroad between nodes k and j; $k \neq j$; $k,j = 1,2,3,4,5$

(Note that all possible combinations of m and n and of k and j do not exist in the diagram.)

Figure 1. Simple Transportation Network

Since a transportation network is used for moving people or material, it is convenient to call the place where the shipment begins the origin and the place where it terminates the destination. The sequence of links of a given transportation type, that begins with an origin node and ends with a destination node, is called a path. This definition of a path is not the standard definition of a path that is commonly used in network and graph theory. An arc is not a path, for a path must include at least two nodes and one arc. In Figure 1, if node 3 is an origin and node 4 is a destination, then the set {(3,3), (3,4), (4,4)} is a path, but the set {(3,4)} is not a path. (1,1) could be a town, at which an iron ore mine is located, and (5,5) could be a seaport. It might be desirable to transport iron ore from the mine to the seaport. Two possible ways of doing this are to go by road from node 1 to node 2 to node 5 or to go by rail from node 1 to node 4 to node 5. Here, node 1 is an origin, and node 5 is a destination. The two paths just mentioned are the sets {(1,1), (1,2), (2,2), (2,5), (5,5)} and {(1,1), (1,4), (4,4), (4,5), (5,5)}. In Figure 1, the set {(1,1), (1,4), (4,4), (4,3), (3,3), (3,5), (5,5)} is not a path for it is composed of both rail and road links and thus is not of a given transportation type. It would be a path if a railroad existed between nodes 3 and 5. One way of avoiding this difficulty and still adhering to the definition of a path is to designate node 3 as an origin and a destination in different paths. Then, the sets {(1,1), (1,4), (4,4), (4,3), (3,3)} and {(3,3), (3,5), (5,5)} are both paths. Thus, a path is homogeneous in a given transportation type. This does not preclude the existence of both a road and a railroad parallel to one another along a path.

The manner of indicating this will become clear later. It has also been noted in the above example that a node can be an origin in one path and a destination in another path.

The planning period for the expansion of the network will consist of T years. At the beginning of the planning period, there may be a number of links in existence. In addition, a finite number of alternative paths for future construction will be given. The total number of existing and possible future paths will be P. The set of links comprising a specific path p will be designated by $L_p$, $p = 1,2,\ldots,P$. In addition, the number of nodes in the network is N. Thus a link $(i,j)$ may have $i = 1,2,\ldots,N$ and $j = 1,2,\ldots,N$, although all possible combinations of i and j may not be economically or technologically feasible during the planning period for the specific country being studied. The set of all links $(i,j)$ that are technologically feasible during the planning period is designated by L. Symbolically,

$$L = \{(i,j);\ (i,j) \text{ is a technogically feasible link in the network during the planning period of T years}\}$$

This set can be obtained from the union of all sets $L_p$:

$$L = \bigcup_{p=1}^{P} L_p.$$

Two other sets that will be convenient for use in the model are the set of all origins S and the set of all destinations $\overline{S}$. Here,

$$S = \{i;\ i \text{ is an origin node}\}$$

and

$$\overline{S} = \{j;\ j \text{ is a destination node}\}.$$

The number of link types in the network is K. For example, a two-lane paved road is a possible link type, as is a one-track railroad. The total number of link types K is divided into four subsets $R_m$, m = 1,2,3,4, where

$$R_1 = \{k;\ k \text{ is a road link}\},$$

$$R_2 = \{k;\ k \text{ is a rail link}\},$$

$$R_3 = \{k;\ k \text{ is a waterway link}\},$$

$$R_4 = \{k;\ k \text{ is an airway link}\},$$

and the four subsets are mutually exclusive and collectively exhaustive, where k is a positive integer that is less than or equal to K. For a given subset $R_m$, m = 1,2,3,4, increasing k implies a more improved link type. For example, consider $R_1$: k = 1 might represent a two-lane dirt road, k = 2 might represent a two-lane paved road, and k = 3 might represent a four-lane paved road. The set $\{k;\ k = 1,2,\ldots,K\}$ and the subsets $R_m$, m = 1,2,3,4, are exogenous to the model and depend upon the objectives of the underdeveloped nation, the state of existing technology, and sources of improvement available to the nation.

B. DECISION VARIABLES AND INDICATOR VARIABLES

For the planning period of T years, a decision must be made to develop certain links in the network during a given year t = 1,2,...,T. The annual budget may preclude the construction of all technologically feasible and desirable links. To assist in arriving at a decision that will maximize the benefits derived from the transportation system, decision variables that are endogenous to the model are needed. The variables will be designated $x_{ijkt}$, indicating whether or not link

$(i,j)$ of type $k$ will be constructed during time period $t$, where $(i,j) \varepsilon L$, $k = 1,2,\ldots,K$, and $t = 1,2,\ldots,T$. These decision variables are integer valued such that

$$x_{ijkt} = \begin{cases} 0 & \text{if link } (i,j) \text{ of type } k \text{ is not constructed in period } t \\ 1 & \text{if link } (i,j) \text{ of type } k \text{ is constructed in period } t. \end{cases}$$

If $x_{ijkt} = 1$, then it is assumed that construction of link $(i,j)$ of type $k$ is begun at the end of period $t - 1$ and completed at the end of period $t$. This assumption need not be restrictive; in practice, construction may begin earlier or later but the cost of the project will be covered by the budget in year $t$ and completed by the end of the period $t$, so that it may be used during year $t + 1$.

Benefits are derived from the network, when paths exist between an origin and a destination. To indicate the existence and operation of a link $(i,j)$ of type $k$ in period $t$, the indicator variable $X_{ijkt}$ will be used and is defined by

$$X_{ijkt} = \sum_{m=0}^{t-1} x_{ijkm}, \quad (i,j) \varepsilon L,\ k = 1,2,\ldots,K,\ t = 1,2,\ldots,T + 1. \qquad (1)$$

Thus,

$$X_{ijkt} = \begin{cases} 0 & \text{if link } (i,j) \text{ of type } k \text{ is not operated in period } t \\ 1 & \text{if link } (i,j) \text{ of type } k \text{ is operated in period } t. \end{cases}$$

Here the variable $x_{ijkt}$ with $t = 0$ was introduced. This is merely a means of specifying the existence of a link prior to the beginning of the planning period. Thus,

$$x_{ijk0} = \begin{cases} 0 & \text{if link } (i,j) \text{ of type } k \text{ does not exist prior to period } t = 1 \\ 1 & \text{if link } (i,j) \text{ of type } k \text{ does exist prior to period } t = 1. \end{cases}$$

## C. BUDGET AND CONSTRUCTION CONSTRAINTS

During time period t, it is assumed that the operating budget is $B_t$, $t = 1,2,\ldots,T$. It is also assumed that the cost of constructing link (i,j) of type k during period t is $C_{ijkt}$ and that the cost of maintaining and/or operating link (i,j) of type k during period t is $M_{ijkt}$. Since deficit financing will not be permitted in the model, the budget for a given period must be greater than or equal to the construction and maintenance/operation costs for the period t. This gives rise to the constraint

$$\sum_{k=1}^{K} \sum_{\substack{(i,j) \\ \varepsilon L}} C_{ijkt} x_{ijkt} + \sum_{k=1}^{K} \sum_{\substack{(i,j) \\ \varepsilon L}} M_{ijkt} X_{ijkt} \leq B_t, \qquad t = 1,2,\ldots,T. \tag{2}$$

To prevent the "construction" of a given link (i,j) of type k in two distinct periods, an additional constraint is necessary, namely

$$\sum_{t=0}^{T} x_{ijkt} \leq 1, \ (i,j) \ \varepsilon L \text{ and } k = 1,2,\ldots,K. \tag{3}$$

For a given (i,j) and k, this constraint will not allow the same link to be constructed twice. For example, one would not desire the model to specify the construction of a two-lane paved road between two nodes in period t = 1 and again in period t = 2. In practice, of course, this would never be done, for it is physically impossible. However, mathematically in the model, without constraint (3), it is possible for two distinct decision variables to dictate the construction of the same link of the same type k in two different time periods.

When $R_m$, $m = 1,2,3,4$, contains two or more elements, then there must be some assurance that only one link of type $k \varepsilon R_m$ exists during any period $t = 1,2,\ldots,T$. Mathematically, this is accomplished with the constraint

$$\sum_{k \varepsilon R_m} \left[ X_{ijk,T+1} - X_{ijkl} \right] \le 1,\ m = 1,2,3,4, \text{ and } (i,j)\varepsilon L. \qquad (4)$$

The $X_{ijkl}$ is present in constraint (4) to permit the links that exist prior to the planning period to be improved. Thus, a two-lane dirt road that exists prior to the planning period can be improved to a two-lane paved road during the planning period. The absence of the $X_{ijkl}$ term in constraint (4) would prohibit the improvement of any link that existed prior to the planning period. Such a situation is not very desirable.

Constraint (4) will, for example, avoid the operation of a two-lane paved road and a four-lane paved road between two nodes i and j during a given period t. It is realized that in the developed countries, where four-lane super highways are constructed, this situation is very possible. However, in an underdeveloped country it is more likely that a two-lane paved road will be converted into a four-lane paved road by adding two more lanes, rather than by constructing four additional lanes. However, since it is conceivable that this latter situation might be desired by the planners in a specific underdeveloped country, then all that must be done is not include constraint (4). When constraint (4) is included in the model, it implies that a link, which is constructed in a given period, will not be improved by additional construction in a later period. Thus, the model will not specify the construction of a two-lane paved road between two nodes in period t = 2 and then the construction of a four-lane paved road between the same pair of nodes in period t = 4. Constraint (4) does not prohibit the operation of both a road and a railroad between two nodes. However, it does make constraint (3) redundant, so constraint (3) may be removed from the model when constraint (4) is included.

## D. OBJECTIVE FUNCTION

As mentioned previously, the purpose of the model is to maximize the benefits derived from the transportation system of an under-developed country. It is assumed that benefits are derived from a path p if and only if all links of the path exist, $p = 1,2,\ldots,P$. Thus, let $W_{pkt}$ represent the benefits derived from path p of type k during period t. The total benefits from the entire period of T years is indicated by the symbol W. The objective function is

$$\text{Maximize } W = \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{p=1}^{P} W_{pkt} \left[ \prod_{\substack{(i,j) \\ \varepsilon L_p}} x_{ijkt} \right]. \tag{5}$$

The bracketed product term is zero if all the links in a given path p do not exist in a period t and is one if all links do exist. At this point in the discussion, no specific explanation is given of the type of benefits that may be maximized. A discussion of benefits will be given in Section IV.

## E. ADDITIONAL REQUIREMENTS

The next consideration in the model is to ensure that information concerning the transportation network that exists at the beginning of the planning period is properly reflected as a constraint. This is accomplished very easily. If link (i,j) of type k exists at the beginning of period $t = 1$, then let $x_{ijk0} = 1$, and $x_{ijkt} = 0$, $t = 1,2,\ldots,T$. Also let $x_{ijmt} = 0$, if $m < k$ and if m and k are both the same type link. Otherwise, $x_{ijk0} = 0$.

Even though it is quite obvious, it is pointed out that, since the notation (i,j) for a link is not an ordered pair, the decision variable for link (i,j) of type k in period t can be written in one of two ways: $x_{ijkt}$ and $x_{jikt}$. The same is true for the indicator variables: $X_{ijkt}$ is the same variable as $X_{jikt}$.

In the basic model, it is assumed that if a link (i,j) is an element of several different paths, then the capacity of the link is sufficient to meet the combined requirements of the common paths. This is a reasonable assumption for it is unlikely that the flow through a given link will be continuous. Thus, this assumption merely generates a scheduling problem to ensure that flow from two different paths in the same direction does not pass through a given link at the same time. This scheduling problem will not be discussed in this paper, but the concept of capacities on links will be considered in Section III.

## F. SUMMARY OF THE BASIC MODEL

A summary of the basic model follows. Constraint (4) is listed, rather than constraint (3):

$$\text{Maximize } W = \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{p=1}^{P} W_{pkt} \left[ \prod_{(i,j) \varepsilon L_p} X_{ijkt} \right]. \tag{5}$$

Subject to

$$\sum_{k=1}^{K} \sum_{(i,j) \varepsilon L} C_{ijkt} x_{ijkt} + \sum_{k=1}^{K} \sum_{(i,j) \varepsilon L} M_{ijkt} X_{ijkt} \leq B_t, \quad t = 1,2,\ldots,T. \tag{2}$$

$$\sum_{k \varepsilon R_m} \left[ X_{ijk,T+1} - X_{ijk1} \right] \leq 1, \quad m = 1,2,3,4, \text{ and } (i,j) \varepsilon L. \tag{4}$$

$$X_{ijkt} = \sum_{m=0}^{t-1} x_{ijkm}, \quad (i,j)\ \varepsilon L,\ k = 1,2,\ldots,K, \text{ and } t = 1,2,\ldots,T+1. \tag{1}$$

$$0 \le x_{ijkt} \le 1 \text{ and integer, } (i,j) \in L,\quad k = 1,2,\ldots,K,\quad t = 1,2,\ldots,T.$$

An efficient algorithm to solve the model is not available, but a solution procedure is presented in Section IV. Since the model is a nonlinear, integer programming model, it may be quite difficult to obtain an efficient solution procedure without adopting a specialized solution procedure, such as a dynamic programming approach. In addition, as the number of links in the transportation system increases and as T increases, the number of decision variables and the number of constraints become very large.

## G. EXAMPLE OF THE BASIC MODEL APPLIED TO A HYPOTHETICAL COUNTRY

An example will now be developed to illustrate the formulation of the model for a specific country. Reference to this example will be made throughout the remainder of the paper.

The current transportation network in the hypothetical country of Levednu is shown in Figure 2. The country currently has an agrarian economy that is basically at the subsistence level. The country grows rice in the region around Ricu and raises cattle in the region of Catvu. At present, very little rice and cattle are exported, although Levednu has the potential for greatly increasing its output of agricultural products. This has been primarily due to a lack of a means to transport the agricultural products for export purposes. Some subsistence fishing is also done along the sea coast in the Blue Sea. Recently, iron ore was discovered in the hills around Oreville. In the light of this situation, the new President of Levednu has directed that a large portion of the annual budget for the next five years be allocated to the Ministry

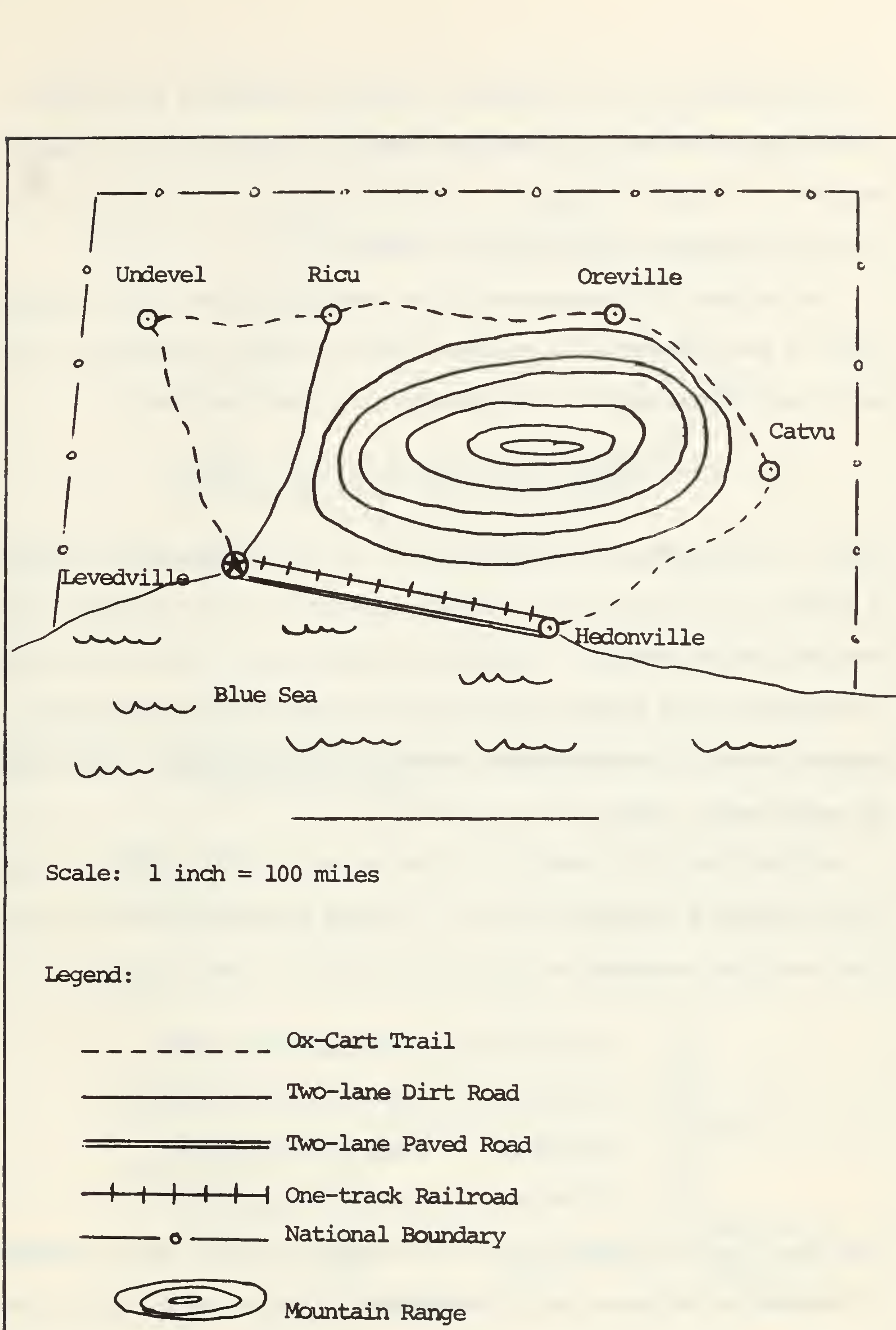



Figure 2. Current Transportation Network in Country of Levednu

of Transportation for the purpose of rapidly developing the nation's transportation system in order that iron ore, cattle, and rice may be exported to foreign markets and to facilitate the distribution of rice and beef throughout the country of Levednu.

The Ministry of Transportation has compiled a list of all desirable links in the transportation network during the next five years. The set of all technoligically and economically feasible links is

$$L = \{(1,1),(1,2),(1,3),(1,6),2,2),(2,3),(3,3),$$
$$(3,4),(4,4),(4,5),(5,5),(5,6),(6,6)\}.$$

Figure 3 illustrates the basic structure of the transportation network. In Figure 3, the arcs do not indicate the type of arc, and cities have been replaced by numbers. Comparing Figures 2 and 3, it is not difficult to determine which numbers replace each city and town, thus becoming numbered nodes. In this example, there are thirteen links: six nodes and seven arcs. Thus, N = 6 and T = 5.

For each arc it is possible to have a road and/or a railroad. Each road is either a two-lane dirt or a two-lane paved road, and each railroad is either one-track or two-track, so K = 4. For the arcs

$$k = \begin{cases} 1 & \text{if the arc is a two-lane dirt road} \\ 2 & \text{if the arc is a two-lane paved road} \\ 3 & \text{if the arc is a one-track railroad} \\ 4 & \text{if the arc is a two-track railroad.} \end{cases}$$

This determines the subsets $R_1 = \{1,2\}$ and $R_2 = \{3,4\}$. This information is supplied to the model and is determined by other means, making the data exogenous to the model. For the nodes, the four values of k represent facilities that are comparable in quality and degree of development to the above values of k for the arcs. For example, k = 4 is a

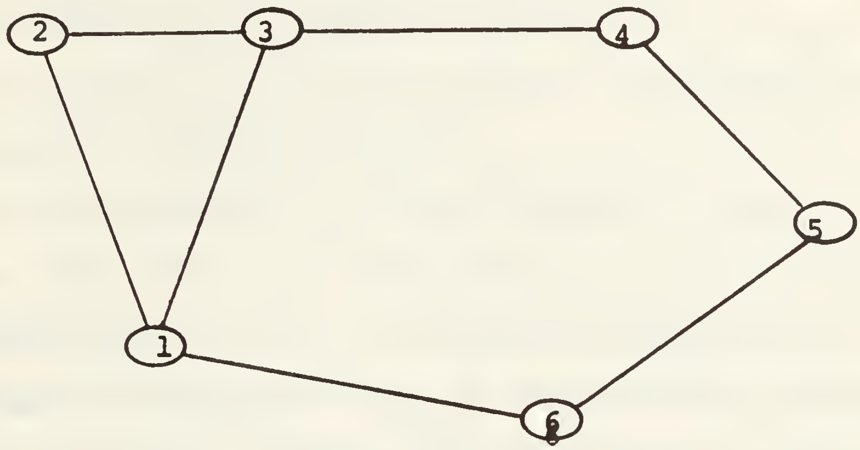
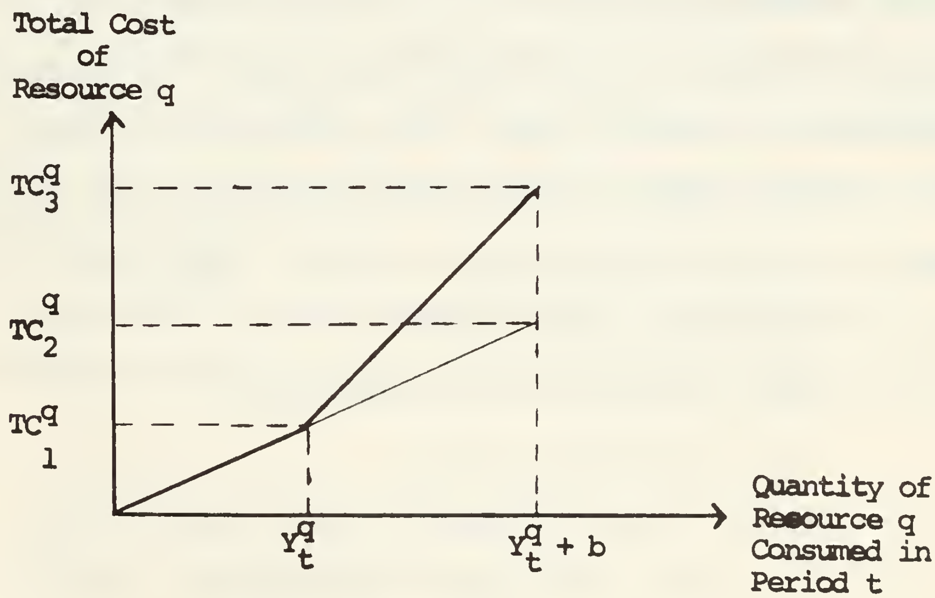



Figure 3. Transportation Network Structure fot the Country of Levednu.




Note: This figure is to be used with Section III, Part E.

Figure 4. Cost of Resources Purchased from International Markets.

better railroad station than k = 3, and its cargo-handling capacity is greater. Such facilities may only exist at origins and destinations in this model. Examining the set L, one notices the absence of arc (1,5). This arc is not economically feasible due to the large mountain that exists between node 1 (Levedville) and node 5 (Catvu), as seen in Figure 2. Arc (1,5) can be included in the model, but its elimination reduces the number of decision variable and constraints, thus simplifying the analysis. The same is true for arcs (1,4), (3,5), (3,6), and (4,6). Arc (2,4) is also absent from the set L. Arc (2,4) is unnecessary, since the route from node 2 to node 3 to node 4 accomplishes the same purpose that a direct link (2,4) would accomplish for a simple network in such an underdeveloped country as Levednu. In a more complex network, an arc such as (2,4) might be very desirable.

The nodes, which are designated as origins and destinations, are also exogenous to the model. The set of origins is S = {3,4,5}, and the set of destinations is $\bar{S}$ = {1,2,3,4,5,6}, which is all of the nodes. This does not imply that each origin-destination combination will be represented in a specific path. This would require at least eighteen paths. In this example, there are fourteen paths in the transportation network, from which benefits may be derived. These are

$L_1 = \{(3,3),(1,3),(1,1)\}$

$L_2 = \{(3,3),(1,3),(1,1),(1,6),(6,6)\}$

$L_3 = \{(3,3),(2,3),(2,2)\}$

$L_4 = \{(3,3),(3,4),(4,4)\}$

$L_5 = \{(3,3),(3,4),(4,4),(4,5),(5,5)\}$

$L_6 = \{(4,4),(3,4),(3,3),(1,3),(1,1)\}$

$L_7 = \{(4,4),(4,5),(5,5),(5,6),(6,6)\}$

$$L_8 = \{(4,4),(4,5),(5,5),(5,6),(6,6),(1,6),(1,1)\}$$

$$L_9 = \{(5,5),(4,5),(4,4)\}$$

$$L_{10} = \{(5,5),(4,5),(4,4),(3,4),(3,3)\}$$

$$L_{11} = \{(5,5),(4,5),(4,4),(3,4),(3,3),(2,3),(2,2)\}$$

$$L_{12} = \{(5,5),(5,6),(6,6)\}$$

$$L_{13} = \{(5,5),(5,6),(6,6),(1,6),(1,1)\}$$

$$L_{14} = \{(5,5),(5,6),(6,6),(1,6),(1,1),(1,2),(2,2)\}.$$

In each path listed, the first link listed in each set is the origin, and the last link listed in each set is the destination. In addition, the order in which the links are listed in each set represents the order of the links in the actual path on the ground from origin to destination. Of course, node 3 (Ricu) is the origin for rice shipments, node 4 (Oreville) is the origin for iron ore shipments, and node 5 (Catvu) is the origin for cattle shipments. Node 1 (Levedville) and node 6 (Hedonville) are port facilities for exporting goods from the country. It is realized that there are many small villages spread throughout the country of Levednu, but these villages do not contribute anything to the analysis and are omitted from the basic structure of the transportation network.

As the transportation system currently exists (see Figure 2), the decision variables for existing links are initialized to the value one. Thus,

$$x_{1120} = x_{1130} = x_{3310} = x_{6620} = 1,$$

$$x_{6630} = x_{1310} = x_{1620} = x_{1630} = 1.$$

This implies that all other decision variables with $t = 0$ are initialized to zero. In addition,

$$x_{111t} = x_{161t} = x_{661t} = 0,$$

since a two-lane paved road exists between nodes 1 and 6; a two-lane dirt road will not be constructed between nodes 1 and 6. Also,

$$x_{112t} = x_{113t} = x_{331t} = x_{662t} = 0,$$
$$x_{663t} = x_{131t} = x_{162t} = x_{163t} = 0, \quad t = 1,2,3,4,5.$$

This merely indicates that existing links of a particular type will not be constructed again during the planning period.

The objective function for maximization is

$$W = \sum_{t=1}^{5} \sum_{k=1}^{4} \sum_{p=1}^{14} W_{pkt} \left[ \prod_{\substack{(i,j) \\ \varepsilon L_p}} X_{ijkt} \right] .$$

The parameters $W_{pkt}$ will not be quantified, but will be discussed in Section IV. The budget constraint is

$$\sum_{k=1}^{4} \sum_{\substack{(i,j) \\ \varepsilon L}} C_{ijkt}\, x_{ijkt} + \sum_{k=1}^{4} \sum_{\substack{(i,j) \\ \varepsilon L}} M_{ijkt} X_{ijkt} \le B_t ,$$

$C_{ijkt}$, $M_{ijkt}$, and $B_t$ are also exogenous to the model and will be discussed in Section IV. The other constraints in the basic model are:

$$\sum_{k=1}^{2} \left[ X_{ijk6} - X_{ijk1} \right] \le 1, \quad \text{for each } (i,j)\ \varepsilon L;$$

$$\sum_{k=3}^{4} \left[ X_{ijk6} - X_{ijk1} \right] \le 1, \quad \text{for each } (i,j)\ \varepsilon L;$$

$$X_{ijkt} = \sum_{m=0}^{t-1} x_{ijkm}, \quad \text{for each } (i,j)\,\varepsilon L,$$

$k = 1,2,3,4$, and $t = 1,2,3,4,5$;

$$0 \le x_{ijkt} \le 1$$

and integer, for each $(i,j)\ \varepsilon L$,

$k = 1,2,3,4$, and $t = 1,2,3,4,5$.

Further reference to this example and the above formulation will be made in Sections III and IV.

## III. SPECIAL CONSTRAINTS AND EXTENSIONS

The basic model, as presented in Section II, is an integer programming model with a nonlinear objective function and linear constraints. In this section some additional constraints are presented to account for special situations and to extend the basic model.

### A. A REQUIREMENT FOR COMPLETE PATHS

As illustrated by the objective function, benefits are only obtained from complete paths in the basic model. A complete path is one in which all links of the path exist. This implies that incomplete paths do not contribute any benefits to the objective function and should be avoided. If it is desired that a path have all or none of its links operating, the following constraint may be added:

$$\prod_{(r,s)\,\varepsilon L_p} \left[\sum_{k \varepsilon R_m} X_{rsk,\,T+1}\right] = \sum_{k \varepsilon R_m} X_{ijk,T+1}, \qquad (6)$$

for each $(i,j)\ \varepsilon L_p$,

$p = 1,2,\ldots,P$, and $m = 1,2,3,4$.

This constraint ensures that all indicator variables in a path are either all zero or all one. Since this constraint is nonlinear and applies to every path in the network, the model becomes more complicated with its addition to the model. Its addition does not alter the procedure in the basic model for selecting complete paths. An alternate way to ensure that incomplete paths do not exist in the transportation network is to add a slack variable to constraint (2) to represent the portion of the budget that is not used for constructing and operating complete paths. This same slack variable is also placed in the objective function with a benefit multiplier that is consistent with a benefit that could be obtained from an alternate investment for that portion of the budget that is not used for complete paths. Here again, the procedure in the basic model for selecting complete paths is not altered.

B. CONCEPT OF CAPACITIES ON LINKS

Although the objective function is intentionally very general at this point, the idea of capacities on the links might be a desirable concept to include in the model. This would not be done if the benefits being maximized by the objective function included similar capacities. One step in this direction is to ensure that the capacities of the links are sufficient to handle the expected flow in the transportation system. Let $v_{ijk}$ be the flow capacity of link (i,j) of type k and let $V_{it}$ be the expected flow from origin i during period t, for each $(i,j)\varepsilon L$, $k = 1,2,\ldots,K$, and $t = 1,2,\ldots,T$. Now, for each origin, the flow out must be less than the capacities of existing links emanating from this origin. Symbolically,

$$\sum_{k=1}^{k} \sum_{\substack{j \ni \\ i \neq j \ \& \ (i,j)\varepsilon L}} v_{ijk}X_{ijkt} \geq V_{it}, \quad i\varepsilon S \text{ and } t = 1,2,\ldots,T. \qquad (7)$$

Next, the capacity at each destination must be greater than the sum of the expected flows from origins that are elements of a path with the given destination. It is realized that a node can be a destination in several different paths. Symbolically, this gives rise to the constraint:

$$\sum_{k=1}^{K} v_{rrk} X_{rrkt} \geq \sum_{\substack{p=1 \\ \text{nodes} \\ r \,\&\, s \\ \varepsilon L_p}}^{P} V_{st}, \quad \text{for each destination } r\varepsilon\overline{S} \text{ and each } t = 1,2,\ldots,T, \text{ where } s \text{ is an origin.} \tag{8}$$

It is also necessary that the facilities at the origins be able to handle the expected flow generated at the origin. Symbolically,

$$\sum_{k=1}^{K} v_{ssk} X_{sskt} \geq V_{st}, \quad \text{for each origin } s\varepsilon S \text{ and } t = 1,2,\ldots,T. \tag{9}$$

In this portion dealing with flow capacities, it is recognized that an arc that is internal to a path may be an element of several paths. Thus, it is possible that a flow stoppage or "bottleneck" could be created at this internal link. To avoid this, in the formulation stage of the problem, one defines this arc with its two associated nodes as another path, the two nodes being designated as either an origin or a destination, as appropriate.

The example that was developed at the end of Section II can be continued to illustrate the use of constraints (7), (8), and (9). Consider node 4 (Oreville), which is an origin. Referring to Figures 2 and 3, for the second year of the planning period when $t = 2$, the constraint (7) becomes

$$\sum_{k=1}^{4} \left[ v_{34k} X_{34k2} + v_{45k} X_{45k2} \right] \geq V_{42}.$$

Here, the parameter $V_{42}$ is the predicted output of iron ore from Oreville (node 4) in the second year of the five year planning period. In addition, $v_{34k}$ and $v_{45k}$ are the arc capacities for arcs (3,4) and (4,5) of type $k = 1,2,3,4$ in the second year of the five year planning period. Since the indicator variables are dimensionless, then these parameters must be in the same unit of measure, such as tons or thousands of tons. It must also be realized that the iron ore mine at node 4 may not be operating in the second year of the planning period, in which case the parameter $V_{42}$ equals zero. Then, the constraint is automatically satisfied, since the terms on the left side are all nonnegative. As before, no numbers are being assigned to the parameters in this part of the example.

Continuing with the second year of the planning period, consider constraint (8) and node 6 (Hedonville), which is a destination in paths p = 2, 7, 12. The origins in these paths are nodes 3, 4, and 5, respectively. The constraint (8) for the second year in the planning period becomes

$$\sum_{k=1}^{4} v_{66k}X_{66k2} \geq \left[V_{32} + V_{42} + V_{52}\right].$$

As above, $V_{32}$, $V_{42}$, and $V_{52}$ are the predicted outputs of origins, 3, 4, and 5, respectively, in the second year, and $v_{66k}$ is the capacity of the facility at node 6 of type k. This constraint ensures that the destination represented by node 6 is capable of handling all output from origins with which node 6 is associated in paths p = 2, 7, 12.

To illustrate constraint (9), the second year of the planning period and origin represented by node 4 will be used again. Here, it is necessary that the transportation facilities at Oreville (node 4) be capable of handling the predicted output from the iron ore mine in the second year, namely $V_{42}$. The constraint is

$$\sum_{k=1}^{4} v_{44k}X_{44k2} \geq V_{42}.$$

For each of the three constraints just listed in the example, it must be remembered that at most two of the indicator variables for each link (i,j) will be nonzero. For example, $X_{34k2}$, with k = 1,2,3,4, will have at least $X_{3412}$ or $X_{3422}$ equal to zero, and at least $X_{3432}$ or $X_{3442}$ equal to zero. In the latter case, this merely is a result of constraint (4), which will only allow either a one-track railroad or a two-track railroad to exist for arc (3,4), but not both.

C. SPECIAL ECONOMIC REQUIREMENTS

As a nation develops economically, it may be necessary that a particular path exist in the transportation network during a given year, so that a certain industry might function. In general, a path p of type $R_m$, m = 1,2,3,4, might be required during time period $t_1$, where $2 \le t_1 \le T$. Here, it is necessary that all links in the path exist and be operational during period $t_1$. To satisfy this requirement in the form of a mathematical constraint, let

$$\prod_{\substack{(i,j)\\ \varepsilon L_p}} \left[ \sum_{k \varepsilon R_m} X_{ijkt_1} \right] = 1, \quad \text{for a given } m = 1,2,3, \text{ or } 4 \text{ and a given } p = 1,2,\ldots, \text{ or } P. \qquad (10)$$

It is recognized that this constraint is extremely restrictive. An example is the case where a railroad is required between Ricu (node 3) and Levedville (node 1) during the fifth year of the planning period. This may result from a trade agreement that requires large shipments of rice from Levedville to a foreign nation in the fifth year of the planning period. The path under consideration is $L_1 = \{(3,3),(1,3),(1,1)\}$, and the set of link types is $R_m = R_2 = \{3,4\}$. Thus, the constraint (10) becomes

$$\left[X_{3335} + X_{3345}\right]\left[X_{1335} + X_{1345}\right]\left[X_{1135} + X_{1145}\right] = 1.$$

This constraint requires that a railroad exist between nodes 3 and 1 during the fifth year of the planning period.

Another special constraint arises when the requirement for a given path p is stated in one of the following ways:

1. The path will exist, but only one type will be present, that is, either $R_1$ or $R_2$ or $R_3$ or $R_4$ in period $t_1$.
2. Not more than one path p will be constructed by period $t_1$. Here it is not necessary to construct the path.
3. At least one type of path p will be constructed by period $t_1$. Now it is possible for the path to exist in period $t_1$ in more than one type.

For the case where the choice is a road and/or a railroad for path p, the constraint becomes

$$\prod_{\substack{(i,j)\\ \varepsilon L_p}} \left[\sum_{k \varepsilon R_1} X_{ijkt_1}\right] + \prod_{\substack{(i,j)\\ \varepsilon L_p}} \left[\sum_{k \varepsilon R_2} X_{ijkt_1}\right] \begin{Bmatrix} = \\ \leq \\ \geq \end{Bmatrix} 1. \tag{11}$$

The =, ≤, and ≥ symbols apply to the three cases listed above, respectively. There are, of course, other possible cases which may be added to above list.

To illustrate this type of constraint, consider the situation which requires at least a road or a railroad between Catvu and Hedonville (nodes 5 and 6) during the third year of the planning period. Here, the appropriate path is $L_p = L_{12} = \{(5,5),(5,6),(6,6)\}$. The constraint is

$$\left[X_{5513} + X_{5523}\right]\cdot\left[X_{5613} + X_{5623}\right]\cdot\left[X_{6613} + X_{6623}\right]$$
$$+ \left[X_{5533} + X_{5543}\right]\cdot\left[X_{5633} + X_{5643}\right]\cdot\left[X_{6633} + X_{6643}\right] \geq 1.$$

It can be seen that a path such as $L_8$, which has seven links, would have a constraint, in which the left side would be the sum of two terms. Each of these two terms would be the product of seven expressions, one for each of the seven links in the path. This type of constraint should be avoided, if possible, since it reduces the set of feasible solutions, greatly increases the nonlinearities in the model, and may limit the benefits that are obtained from the transportation system as compared to the benefits that might be obtained if the constraint were absent. Such a constraint might also make the solution of the model trivial.

D. POLITICAL CONSTRAINTS

The next special constraint is a "political" constraint. It may be required that a given link be constructed before another link. In general, the constraint requires that link (r,s) of type $k_1$ be constructed before link (u,v) of type $k_2$, where $k_1$ may or may not equal $k_2$. Mathematically, this is written as

$$X_{rsk_1t} \geq X_{uvk_2t}, \quad t = 1,2,\ldots,T + 1. \tag{12}$$

It must be remembered that if there are too many constraints of this type, the model becomes very restrictive. There may only be a single solution, which requires very little analysis and may defeat the purpose of using a model such as the one in this paper. Used carefully, though, constraint (12) can add realism to the formulation.

Using the same example to illustrate this constraint, assume that it is directed that an improved railroad station be constructed at Hedonville (node 6) before a two-track railroad is constructed between

Catvu (node 5) and Hedonville. Since Hedonville already has a railroad station that will accommodate a one-track railroad ($x_{6630} = 1$), then it is recognized that $k_1 = k_2 = 4$, $(r,s) = (6,6)$, and $(u,v) = (5,6)$. The constraints are

$$X_{664t} \geq X_{564t}, \quad t = 1,2,3,4,5.$$

E. CONSTRAINTS ON MATERIAL RESOURCES

One special situation that can be expected to arise in an underdeveloped country is a limited supply of specific resources in particular periods. For example, a limited number of bulldozer operators may be available during a given year from the nation's own labor supply. Once this number is exceeded, it will be impossible to obtain additional bulldozer operators without obtaining them from an international labor supply at a much higher cost than is paid to local operators.

In general, let $Y_t^q$ be the quantity of resource q that is available for use in period t from national sources of supply, where $q = 1,2,\ldots,Q$ and $t = 1,2,\ldots,T$. Next, let $y_{ijkt}^q$ represent the quantity of resource q that is required to construct link $(i,j)\varepsilon L$ of type $k = 1,2,\ldots,K$ during period $t = 1,2,\ldots,T$. The constraints on material resources are:

$$\sum_{k=1}^{K} \sum_{\substack{(i,j) \\ \varepsilon L}} y_{ijkt}^q x_{ijkt} \leq Y_t^q, \quad q = 1,2,\ldots,Q, \text{ and } t = 1,2,\ldots,T. \tag{13}$$

If the underdeveloped country decides that it will purchase additional quantities of resource q from international supplies, then this constraint is no longer applicable. Then it is necessary to modify constraint (2), the budget constraint, since the added cost will be funded from the annual budget $B_t$. This is accomplished by adding another cost term to the left side of constraint (2) for each resource purchased from international sources to reflect the additional cost thus incurred. This can be seen in Figure 4, which was given previously in Section II, where total cost of resource q is plotted against the quantity of resource q that is used during a given period. If $Y_t^q$ is the quantity used, then the total cost of resource q is $TC_1^q$. However, if b additional units of the resource are used, then the total cost is $TC_3^q$. If the total requirement could be supplied by the underdeveloped country, then the total cost would be $TC_2^q$. However, since this is not possible, the difference in cost that results from the purchase from the international market is $TC_3^q - TC_2^q$. Thus, the new cost term that would be added to constraint (2) would be dependent upon the decision variables that were equal to one, that is, decision variables for links that would be constructed, and to the additional total cost incurred for each resource obtained from the international market.

Inherent in this process of obtaining a scarce resource from the international market is an important decision. If it is possible to develop the resource within the underdeveloped country, such as training additional bulldozer operators, then the decision should be made to develop the resource if the cost of the development is less than $TC_3^q - TC_2^q$. This is the additional total cost incurred for each resource q obtained from the international market. It is assumed that such a development will not delay the project(s) for which the resource is needed. If there is an unacceptable delay, then this development of the resource by the nation will not be undertaken. It is recognized that the decision that must be made is not a simple one, but it is an alternative that must not be overlooked. No detailed discussion will be given, nor will a decision rule be presented. The likelihood of such a situation is merely mentioned in passing.

## IV. DISCUSSION AND CONCLUSIONS

### A. BENEFITS --THE OBJECTIVE FUNCTION

The objective of the model is to maximize the benefits derived from the transportation system of the underdeveloped country. In order to accomplish this, equation (5) was given in Section II. The word benefit is a general term until it is clearly defined, and even then there may be some confusion. It is the purpose of this portion of the paper to explain the meaning of the term benefit as it is used in the objective function of the basic model.

Since the model that has been developed is a very general one, each application of the model to a specific country will require certain modifications to the model presented in this paper, in order to describe accurately the situation in that country. This is especially true for the parameters $W_{pkt}$ in the objective function. The meaning of these parameters in a specific country will depend largely on the goals of the country and the purpose for which the transportation system is being expanded.

Most nations set up a number of goals for their economic development programs. Ackoff [1] gives four general examples: (1) To decrease the disparity of incomes; (2) to decrease unemployment to a low level; (3) to improve the standard of living; and (4) to become economically independent of other nations. Of course, these goals may have conflicting criteria when the country actually attempts to achieve them. Chenery and Kretschmer [4] suggest discussing an underdeveloped country in terms of welfare functions. They point out that, in practice, it is quite difficult to quantify and/or measure a welfare function and that one may be forced to use a measure of welfare that is the total availability of material goods and services that can be used for consumption and investment for public and private purposes. Then the goal is to maximize the welfare of the country.

In discussing benefits that are derived from projects, McKean [14] uses the terms primary and secondary benefits. He defines primary benefits as the value of the immediate goods and services that result from a project, and secondary benefits as the values added over and above the value of the immediate goods and services. He does point out that it may be quite difficult distinguishing between the two and that problems of measurement and over-counting may be expected to arise. A similar approach will be taken to measure and quantify the benefits derived from an expanding transportation system in an underdeveloped country.

A departure in this direction is to classify benefits as direct and indirect. In attempting to define direct and indirect benefits, one discovers that as the definition becomes more specific, the coverage of the definition becomes more restrictive. It is for this reason that general definitions will be used. A direct benefit is an immediate gain, favorable change, or improvement that can be measured and quantified in the region of the country in which the change in the transportation network takes place and that can be shown to have been caused by the change in the transportation network. An indirect benefit is a gain, favorable change, or improvement that is derived from a change in the transportation network, but which may not be immediately obvious and for which direct causality may not be apparent or evident. It is hoped that these definitions will become clearer as the discussion proceeds.

In the discussion that follows, some examples of direct and indirect benefits will be given. It is not intended that these examples be exhaustive of the possible alternatives or that they be non-overlapping. In fact, some of the examples are not even compatible with one another, so it may be difficult to use a combination of them in the objective function of the basic model. They are merely candidates for selection. McKean [14] uses another term to describe the measurement problem. He calls intangibles any consequences of the compared alternatives that cannot be translated into the common denominator being used. Such consequences may fall into the grey area of classification into the areas of direct and indirect benefits. With these thoughts in mind, some examples will now be discussed.

One direct benefit may be an increase in the production of agricultural products or mineral deposits that resulted from the opening of new farmlands or new markets, which would not have been possible without the changes in the transportation system. This can be quantified in terms of volume of goods shipped, profit made by the farmer or by the producer, or more generally, as a change in the national income of the country.

A different approach is to examine a change in the opportunity cost. Thus, a direct benefit is the change in the cost of transportation that results from an improved transportation network. The volume of goods transported times the decrease in the cost of transportation will be a benefit that will go to the consumer and/or to the producer as a reduced cost or a larger profit. [18] For the consumer, a particular product, which was previously too expensive or unavailable for purchase, may be attainable. Here the producer will receive a profit, whereas the consumer derives an increased amount of satisfaction from the availability of the product. It may not be possible to assign a cardinal value to this derived satisfaction of the consumer.

A third area for investigation is the possible increase in the number of business and commercial establishments in the region of the transportation network change. It may be difficult to classify this as either a direct or an indirect benefit. It can be quantified in terms of the increase in national income that may result, but it may be difficult to show direct causality to the change in the transportation network. In the example of the country of Levednu, the iron ore mine at Oreville is virtually useless without a transportation system for shipping the iron ore. As both are developed, it can be expected that the town of Oreville will grow in size and in economic activity. This growth can be measured and quantified, but it is not possible to attribute the growth solely to the iron ore mine or to the transportation system. This is a good example of the difficulty that is encountered when one begins to list various alternative candidates for use in the objective function as benefits.

The direct benefits can be expected to be more difficult to evaluate and may not take place immediately. In fact, knowledge that the new transportation system was partially or wholly responsible for a particular benefit may be lacking. In addition, the actual measurement of the benefit may not be possible even if the benefit is associated with the change in the transportation system.

One indirect benefit may result from the increased markets that become possible with transportation system improvements. Although the markets themselves will produce direct benefits in increased sales and volume, many side effects may be generated. Examples are: (1) Ideas for additional economic activity in other parts of the country may be created; (2) knowledge of better techniques of production may be obtained; and (3) the unemployed may learn about new employment opportunities that the nation's expanding economy is providing. These side effects may be difficult to measure or may fall into the category of intangibles. In addition, they may provide benefits in later times or in distant regions from the original change in the transportation network.

Another benefit may be a decrease in dependence upon imports as a major source of supply for many products. Depending upon the situation, it may be a direct or an indirect benefit. As transportation costs decrease, it is possible that locally produced goods may become competitive with imported items in the market. This can lead to an increase in the production of the local product and the eventual exporting of the product. If this occurs, the desire of foreign investors to finance future projects in the underdeveloped country may increase. It must also be realized that the opposite effect may take place, that is, imported goods may become less expensive. This may be interpreted as a benefit as long as no adverse effects are created in the local economy. In either case this type of benefit is possible, but the true cause may not be apparent.

The desire to decrease unemployment was mentioned previously. Krause [11] points out that labor as a whole is generally used poorly in underdeveloped countries, that is, it yields low productivity. This is not due as much to unemployment as it is to what Krause calls underemployment, or the inefficient use of labor. Underemployment can exist for any factor of production, but it is most commonly associated with labor. As an illustration of the idea of underemployment, consider a farmer and his son who produce a crop such as rice. If a second son joins the farmer and the first son but the output does not increase, then marginal product of the additional laborer is zero, and this laborer is underemployed.

Since most underdeveloped countries are raw-material producers, employment tends to be concentrated in agriculture or raw-material production with little emphasis or expertise in manufacturing. This concentration of the labor force can be attributed to three important factors. First, output per worker is low, so that in the area of food production many persons are required to produce the minimum necessary food supplies for subsistence. Second, the pattern of production is aimed toward exporting raw materials. Third, the general absence of alternatives for employment is sufficient reason to perpetuate the underemployment situation. Thus, a properly expanded transportation network can greatly reduce the underemployment situation by opening up alternate employment opportunities of a productive nature. This can result in higher levels of real per-capita income and in improved living conditions for the people. Hopefully, not only will the level of real national income increase, but it will rise cumulatively over a period of time, the increase will benefit the entire population rather than just a small minority, and economic development will be the end result. [11] Although these results can be quantified, it is difficult to specify which segment of the transportation system made them possible. It is also difficult to separate the effects that other segments in the economy may have had in producing these benefits. These benefits would not be immediately evident, either. For these reasons, these benefits can be better categorized as indirect rather than direct.

The social benefits that are derived by an underdeveloped country from improvements in the transportation system are usually indirect benefits, although the distinction may not be obvious in each case. Improved roads can make it possible to operate schools, health clinics and hospitals, and entertainment and recreation centers, that could not otherwise be operated and be accessible to the general populace. As the quality and quantity of food that is available to the consumer increases, it can be expected that the infant mortality rate will decrease and individual life expectancy will increase. With more schools, literacy rates will increase, and more individuals will become qualified to work in jobs that require skilled labor. Infant mortality rates, life expectancies, and literacy rates can be measured and quantified, but it may be impossible to express them in terms of a common denominator. More importantly, it might not be possible to determine what influence the changes in the transportation network had in creating these social benefits, yet alone what portions of the network were responsible for them. It is for these reasons that these social benefits are called indirect.

As the transportation network in a country develops, it is likely that government officials will be able to travel throughout the nation and reach the majority of the populace. This may contribute to the growth of a spirit of nationalism. Although one can measure the amount of traveling of government officials, it is not possible to measure the spirit of nationalism or to determine whether changes in the transportation network actually made a substantial contribution to this growth of nationalism. Similarly, the expansion of the transportation network can do much to strengthen the internal security of the underdeveloped nation, for the police and military will have greater mobility. The problem of measuring this change in internal security is quite difficult. Therefore, the above benefits are indirect. In both cases, the changes may not even be evident to an experienced observer, for the spirit of nationalism and the strengthening of the internal security of the nation may not be shown until a future date.

Now that some of the potential benefits have been discussed, the very difficult problem that remains is the proper selection of the direct benefits to be maximized in the model. Hitch and McKean [7,14] treat this topic in some detail. The benefits selected must be expressed in terms of a common denominator, or in the same unit of measure. Overcounting or double counting must be avoided. Next, it is possible that a benefit from one project may be a cost to some other segment of the economy, so adverse spillover effects must be considered in selecting benefits for the model. It is also necessary to be able to predict with accuracy what the expected benefits from a given improvement in the transportation system will be. Since the model covers a period of T years, the benefits obtained from the existence of a particular path $p = 1,2,\ldots,P$ at any time during the period of T years must be known with a reasonable degree of certainty or confidence. Although it is not the purpose of this paper to discuss this point, it must be pointed out that the experience of other nations and the historical results of the country under study can provide important and valuable information, upon which to base statistical prediction studies. The particular statistical tools that will be relevant will vary from country to country based on the situation and information available and the length of the planning period, to mention just a few.

Returning to the example of the country of Levednu, it is recognized that the goals of the development program are to increase the country's exports of iron ore, rice, and cattle, and to increase the distribution of rice and beef throughout the country. The main benefit, which will be obtained from this program, is the revenue and the resulting profits that are generated from the increased agricultural and mine products that are shipped along the links of the transportation network. Thus, $W_{pkt}$ will represent the profit obtained from shipping a given volume (tonnage) of iron ore, rice, or cattle along path p of type k during year t, where p = 1,2,...,14, k = 1,2,3,4, and t = 1,2, 3,4,5. A necessary input will be the predicted amount of a particular product that can be generated for shipment along a particular path during a given year. Thus, if it is predicted that 10,000 tons of rice can be grown for shipment from Ricu to Levedville for export during the year t = 2, then one needs the parameters $W_{112}$, $W_{122}$, $W_{132}$, and $W_{142}$. These parameters will have the value that is obtained from multiplying the profit per ton times the number of tons that can be shipped along the given route type k. Of course, this profit equals revenue minus costs, and included in the costs are the costs of physically transporting the goods along the type path under consideration. Thus, it is unlikely that each of the four listed parameters will have the same value. Similar calculations would be made for each of the other thirteen routes in the proposed transportation network. In this example a simple criterion was selected, so the problems of suboptimization must be considered. These will be discussed later in this paper.

Since the benefits are being summed in the objective function for different periods of time, the benefits will be discounted values. The determination of the discount rate is not a problem that is peculiar to the transportation system analysis, for a discount rate will be needed in other segments of the economy where project planning spans several time periods. Although the determination of the discount rate may be a problem, its application to this model is not difficult. A simple example will illustrate this. Let i be the common discount rate for the period of T years. If $W^*_{pkt_1}$ is the true value of the benefits derived from path p of type k in the year $t_1$ and $W_{pkt_1}$ is the present value of the benefits (discounted value), then

$$W_{pkt_1} = \frac{W^*_{pkt_1}}{(1 + i)^{t_1 - 1}} \qquad (14)$$

A more complete treatment of this subject may be obtained from Chapter 18 of Baumol [2]. However, one must realize that Baumol defines the discount rate as $1/(1 + i)$ rather than i.

## B. EXOGENOUS PARAMETERS

Constraint (2), which includes the annual transportation budget, contains input parameters. The values of the $B_t$, $t = 1,2,\ldots,T$, are determined outside of the model by the underdeveloped country's budgeting process, which will not be discussed in detail in this paper. It must be realized that the values of the $B_t$ are only a portion of the annual budget of the agency in the underdeveloped country that is responsible for developing the transportation system. Other expenses, such as the administrative costs of operating the agency, must be met from its total budget. However, these costs are not considered in this model, and the value of $B_t$ for each year is considered to be the working budget for expanding and maintaining the transportation system in the country. Thus, one decision that must be made outside of the model is the determination of the portion of the transportation agency's total budget that will be devoted to the expansion and maintenance of the transportation network. It is assumed that the major portion of the total budget will be so allocated.

The actual cost of constructing each link, $C_{ijkt}$, and the actual cost of maintaining existing links, $M_{ijkt}$, are also important input parameters to constraint (2). If the estimates of these parameters are grossly in error for portions of the network, then the results that are predicted by the model will not be achieved. The problem becomes more acute as the length of the planning period of T years increases. Although the values of these parameters are highly dependent upon the existing technology for constructing the various links in the transportation network, one major problem is the ability to estimate the cost of labor throughout the years in the planning period. Another problem deals with the estimation of the cost of the necessary material resources and their availability during the period. Thus, since the parameters $C_{ijkt}$ are the total cost of constructing a link (i,j) of type k during period t, the actual estimation problem might necessitate a separate study. The same is true of the parameters $M_{ijkt}$. In addition, the cost of constructing a mile of link (r,s) of type $k_1$ might not be the same as the cost of constructing a mile of link (u,v) of type $k_1$. This can be best illustrated by means of an example using the country of Levednu. Since a two-lane dirt road presently exists between Ricu and Levedville (nodes 3 and 1, respectively), the cost per mile of constructing a two-lane paved road on link (1,3) might be less than the cost per mile of constructing a two-lane paved road between Undevel and Levedville on arc (1,2), since only an ox-cart trail exists between these nodes. It is reasonable to assume that the cost of improving an ox-cart trail would be greater than the cost of improving a two-lane dirt road, as long as there are no unusual differences between the two links, such as the existence of a drainage problem on the link of greater present development.

Although this model is not a formulation using conventional network and graph theory, when one begins to use capacity constraints on the links in the network, as is done in constraints (7), (8), and (9), then a problem is encountered that is similar to that encountered in the maximum flow problems of network and graph theory. The problem is the estimation of the flow capacity of a link of a given type in the network, namely $v_{ijk}$. Before this can be accomplished, it is necessary to determine the unit of measure of flow. This will depend, of course, upon the commodities being transported through the network. Historical data and experiences of a similar nature in other nations will be extremely important. Not only will the type of transport being used be important, but the efficiency with which it is being used is critical. One can only assume that existing and potential transport means will be used efficiently. As before, in estimation of the construction and maintenance parameters, a detailed study will be necessary to determine the values of link capacities, if capacity constraints are used in the model. The expected outputs from the origins, $V_{it}$, $i \varepsilon S$, are also necessary inputs when the capacity constraints are used. However, these inputs are normally obtained from other departments, ministries, or agencies. For example, in the country of Levednu the quantity of rice that will be available for shipment from Ricu in the year t = 2 will be obtained from the Ministry of Agriculture, whereas the amount of iron ore that is expected to be available for shipment from Oreville in the year t = 2 will be obtained from the ministry that is responsible for the mining of iron ore, or from the private firm that is operating the mine. The fact that the parameters $V_{it}$ are being obtained from sources outside of the transportation agency that is responsible for developing the network does not lessen their importance or criticality in the formulation and analysis of the problem.

Finally, when constraints on material resources are used in the model, it becomes necessary to obtain estimates of the parameters $y^q_{ijkt}$ and $Y^q_t$, the quantity of a given resource needed to construct a particular link and the total quantity of that resource available within the underdeveloped country, respectively. As was pointed out in Section III, when it is decided to purchase additional quantities of a given resource from international supply sources, then it is necessary to modify the budget constraint. Thus, accurate estimates of the availability of a given resource in a particular time period is of utmost importance. As before, a detailed study of this problem will be necessary so that the input parameters $y^q_{ijkt}$ and $Y^q_t$ are not grossly in error.

Although the techniques for estimating exogenous parameters have not been discussed, it is apparent that this area is one that must be given major consideration, for the success or failure of the planning of the expansion of the transportation system is highly dependent upon these estimates. This is an area in which additional study is warranted.

## C. SOLUTION TO THE MODEL

As the basic model is presently formulated, it is a nonlinear model with 0-1 variables, making it an integer programming model. An efficient algorithm to solve the model is not available, so another approach is necessary. The basic difficulty in reaching an optimal solution is the fact that the objective function consists of the sum of products of sums of integer variables. This means that the terms in the objective function consist of products of the decision variables $x_{ijkt}$.

An initial approach to change this situation was an attempt to reduce the objective function to one with separable functions of each of the decision variables and then to use approximating methods to solve the model, as suggested by Hadley. [5] This approach severely complicates the model by introducing a very large number of new constraints and variables, so it will not be discussed.

The number of links in the model is finite and is merely the number of elements in the set L. For the purposes of discussion, call this number n. For any link (i,j) at the end of the planning period, at most four transportation types may exist, namely a road, a railroad, a waterway, and an air route. Thus the maximum number of decision variables $x_{ijkt}$ that have the value one is 4n, although an optimal solution may contain considerably fewer decision variables with the value one. The total number of decision variables in the model is $D = nKT$.

Next, let $r_m$ be the number of elements in the set $R_m$, $m = 1,2,3,4$, such that $K = r_1 + r_2 + r_3 + r_4$. For a given link $(i,j)$ of type $k_1 \varepsilon R_m$, the link may be constructed in any of T different years or not constructed at all, giving $T + 1$ choices. But there are $r_m$ elements in $R_m$, so this really gives $r_m T$ choices for the construction of link $(i,j)$ of type $k_1 \varepsilon R_m$ plus an additional choice if it is not constructed at all, for a total of $(r_m T + 1)$ choices. Now, $m = 1,2,3,4$, and at the end of the planning period a given link $(i,j)$ may exist in each of the four possible types, so the total number of choices is

$$\sum_{m=1}^{4} (r_m T + 1) = (r_1 + r_2 + r_3 + r_4)T + 4 = KT + 4.$$

Since there are n links in the network, the total number of choices is

$$A = (KT + 4)^n. \tag{15}$$

This is the number of solutions to the model that may be physically and practically constructed on the ground. Granted, all of them may not be feasible. As n, K, and T increase, the number A also increases.

As an illustration, consider the example of the country of Levednu. Since the set L has thirteen elements, then $n = 13$. Also, $K = 4$ and $T = 5$. Since there are no waterways or airways in this example, the maximum number of decision variables that have the value one is $2n = 26$ rather than $4n = 52$. The number of solutions is

$$A = [(4)(5) + 4]^{13} \approx 8.77 \times 10^{17}.$$

This very large number can be reduced by taking advantage of the information on existing links in the transportation network, but the reduction does not change the general magnitude of the number A.

It would be foolhardy to consider solving such a problem using hand methods. The use of a high speed computer is needed. Although the present generation of computers are able to solve some very large problems, one must look to future computer technology in both the hardware and software areas, in order to enable one to obtain the solution to such a large problem.

To this point, no mention has been made of the number of constraints in the model as formulated in Section II. The reason for this is that the number of constraints will vary with each formulation. Considering only the basic model for the example of Levednu, there are 31 constraints, plus the requirement that the 260 decision variables be integer 0-1.

Although an efficient algorithm has not been presented to solve the model, a solution procedure will now be outlined. The technique is a lengthy numerical evaluation of each point in the solution space. It is hoped that future high speed computers will allow the multitude of evaluations, that will be suggested below, to be completed in a reasonable length of time that is operationally economical.

The procedure is as follows:

1. Determine all points in the solution space. There are A of these points, as shown by equation (15). Each point can be represented by a vector of the total number of decision variables $D = nKT$, where, at most, $4n$ have the value one, and the remainder are zero.
2. Determine the value of the objective function that corresponds to each point in the solution space found in step (1). This is a straight-forward numerical evaluation of the objective function for each point in the solution space.
3. Next, arrange the numerical values of the objective function that were calculated in step (2) in decreasing order of magnitude.

4. Using the order of numerical values from step (3), select the largest and determine whether or not the point in the solution space, which produced this value of the objective function, is feasible. This is accomplished by substituting this point into each constraint. If each constraint is satisfied, then the point is feasible, and an optimal solution has been obtained. If each constraint is not satisfied, the point is not feasible, and one proceeds to step (5).
5. Select the next largest value of the objective function that was calculated in step (2) and ordered in step (3). Determine whether or not the point in the solution space, which produced this value of the objective function is feasible, by substituting this point into each constraint.
6. If the solution is feasible, the solution is optimal, and the evaluation is terminated. Otherwise, return to step (5).

The main drawback in this procedure is that it is quite lengthy and may require a large amount of time to complete. Also, alternate optima are not obtained, unless each point in the solution space is evaluated for feasibility. Since there are a finite number of points in the solution space, the procedure will converge to a solution, although the convergence may be very slow.

In the event that there is no feasible solution to the problem, then the structure of the problem must be reviewed. It may be necessary to increase the budget, or alternatively, certain of the constraints might be relaxed. In addition, it might be necessary to re-evaluate the development objectives of the transportation system expansion program.

## D. SUBOPTIMIZATION

A very important question that must be asked in any government development program is whether or not the objectives of the program are compatible with the objectives of programs of higher and lateral headquarters. This is not an easy question to answer. Hitch and

McKean [7A] discuss this problem in the context of the difficulties of criteria selection and suboptimization. They define full optimization as the simultaneous consideration of all possible alternatives and the allocation of resources among these alternatives, to include the consideration of the effect of exogenous events, in order to maximize the function of the optimizer. They explain that suboptimization is the process of considering only a few alternatives and only a few allocations of resources among these alternatives in maximizing an objective function. The criterion that is used may be imperfect or inconsistent with those of higher levels. This occurs because only a few assumptions about uncontrollable events can be made. Even when the appropriate criterion is selected, there is often the problem of deciding which costs should be included. If the criterion is the maximization of benefits minus costs of some type, then the selection of the proper benefits is an important decision. In extremely large and complicated problems, suboptimization may occur because important benefits are omitted from the objective function because they are not adaptable to quantitative analysis. [7A] As mentioned previously, the spillover effects cannot be ignored, for the optimization of a benefit in one developmental program may result in an unacceptable cost in another program.

It should be noted at this point that the organizational structure for this transportation system optimization problem is taken as given, and the organizational partitioning problem is not discussed. Here the optimization is being conducted within the transportation system. An alternative organizational structure is the division of the underdeveloped country into regions. Then the economic development in each region is optimized, and the effect of the transportation system changes in each region are only a part of the developmental problem.

In the model that has been formulated in Sections II and III, there is the danger of suboptimization when the model is applied to a specific country or region. Since a budget is given to the agency responsible for expanding the transportation system, the allocation of resources may not be consistent with that of higher levels. This is one area, in which the optimizer must proceed with caution. Next, coordination with higher levels is imperative, when the decision for the selection of an objective function is made. Even though the selection of a criterion for the maximization of benefits may not be entirely consistent with the criteria of higher levels, the degree of inconsistency may be considerably reduced through coordination.

One reason why only a few alternatives are considered is that all alternatives that are relevant may not be known by the optimizer. This is another important reason for close coordination. In conjunction with this, there is uncertainty associated with any planning and allocation function that extends several years into the future. As was pointed out in the discussion of benefits, there are direct and indirect benefits associated with the expansion of the transportation system of an underdeveloped country. It may be true, but not realized by the planner, that certain indirect benefits are extremely important and may be cause for a suboptimization when not considered. Even when the effect of important indirect benefits is realized by the planner, their inclusion in the model may be difficult for it may not be possible to express them in terms of the common denominator being used. Thus, a suboptimization results.

The important point is that the planner must be aware of the dangers of suboptimization and must make a pointed effort to use criteria that are consistent with higher and lateral levels. There is no easy solution to the problem of suboptimization.

One means of reducing suboptimization is through the use of post-optimization studies. One question that arises in a model such as the one in this paper is: What is the effect on the objective function of increasing the budget in the year $t_1$ by an amount b? In a linear program, this type of question can be answered very easily by solving for the dual variable that is associated with $B_{t_1}$ in the dual objective function. In effect, this dual variable gives the change in the objective function of the primal problem for a unit of change in the parameter $B_{t_1}$. See Baumol [2] for a more detailed discussion of the interpretation of the dual variables. However, the model that has been formulated in this paper is not linear, but is nonlinear. Lancaster [12] suggests a technique for writing the dual of a nonlinear problem, in which the objective function and constraint functions are continuous. This technique is not applicable here, for the decision variables are integer 0-1 variables.

An alternative is available, but requires considerable additional calculations. The procedure is to increase the parameter being changed by the amount of the change. Then the numerical evaluation that was previously outlined may be used to resolve the problem. If the parameter being changed is not in the objective function, then only steps (4) to (6) must be repeated. The two numerical values of the objective function so obtained can then be compared, and the ratio of their difference to the change in the parameter being varied will give the change in the objective function due to a unit change in the parameter. In the case of the budget change, this would be

$$\frac{W_2 - W_1}{b} . \tag{16}$$

Here, $W_1$ is the value of the objective function before the budget was increased, and $W_2$ is the value of the objective function after the budget was increased.

The utility of such a ratio arises from the fact that it is very likely that during the planning process, the government may find it necessary to increase or decrease its overall budget. Then the question arises as to which programs will have their budgets increased or decreased. If the same type of objective functions are being used by several programs, then the ratio in (16) may be used to assist in answering this question. Unless there is some other overriding reason, the increase would go to the program that showed the greatest increase in its objective function, and the decrease would be from that program that showed the smallest marginal change. If such a budget increase or decrease is quite large, then it might be necessary to spread it over several programs. It must be emphasized that this comparison is not valid unless similar criteria and objective functions are being used in the programs being compared. In addition, it should be pointed out that a small increase in the budget for the model formulated in this paper may produce no change in the objective function, whereas a decrease might produce a change. This is due to the fact that the variables are not continuous, but are integer. Thus, through the use of post-optimization studies, the government may improve the allocation of its scarce resources. It should be noted that similar procedures can be used to investigate changes in other parameters in the model. It is granted that an efficient algorithm is needed before this procedure may be used freely and economically.

## E. CONCLUSIONS

The model formulated in this paper is a general model that is not to be applied to a specific country or region without suitable modifications to meet the situation of the country, in which the model is being used. The intention was to formulate a model that took advantage of the use of integer 0-1 decision variables to determine the manner in which the transportation system of an underdeveloped country or region is expanded during some finite period of time in the future. This was accomplished using an objective function consisting of the benefits derived from existing paths in the transportation network during each year of the planning period. It was pointed out that considerable research and study would be necessary in order to obtain values for the exogenous parameters that truly reflect the actual physical situation in the country. One must also realize that the model does not make decisions, but it presents to the planner a tool that may greatly assist him in the process of allocating resources, while expanding the transportation system of his country. This is true of most studies and models of this nature. Finally, it must be pointed out that although an efficient algorithm was not obtained for solving the model, this formulation is a step in the proper direction of providing planning tools for the decision maker in underdeveloped countries, so that the allocation of scarce resources may be completed economically and effectively.

## V. FUTURE RESEARCH

The model that has been developed in this paper is intended to assist underdeveloped countries in expanding their transportation systems. It is actually only one step in that direction. As was pointed out in the preceding sections, additional research in this area is not only necessary but very important. This section will briefly sketch some suggested areas for future research.

One of the most pressing problems is the absence of an efficient algorithm to solve the model that has been formulated. Since the model has many variables, that extend over several time periods, the techniques of dynamic programming may prove to be rewarding in this regard.

A related problem to that of finding an efficient algorithm to solve the model is the problem of developing techniques for conducting efficient post-optimality studies or sensitivity analyses.

Next, it would be very helpful if explicit procedures were developed to revise the problem formulation in the event that no feasible solution is found for the model as initially formulated.

The model presented in this paper depended heavily upon many exogenous parameters. The assignment of proper values to these parameters is a very important prediction problem. Thus, it would be very helpful if known techniques for estimating these parameters were cataloged and suggestions were made concerning the applicability of each technique to various situations that might exist in underdeveloped countries that were expanding their transportation systems.

The next step in the development of the model is to allow a link (r,s) of a type $k_1$, which is constructed in year $t_1$, to be upgraded to a type $k_2 > k_1$ in the year $t_2 > t_1$, where a fixed period of time must elapse before the improvement may be made. This capability does not presently exist in the model, but is a desirable feature when the number T is very large, that is, for long-range planning of transportation systems.

The final extension of the model that would facilitate its use in national development programs is the inclusion of the model into a larger system of similar sub-models for resource allocation along the lines of the decomposition procedure of linear programming.

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13. ABSTRACT

A nonlinear integer programming model for expanding the transportation system of an underdeveloped country is presented. The model uses integer 0-1 decision variables. The basic model has linear constraints and a nonlinear objective function. Some special situations and extensions to the model are presented. The benefits being maximized in the objective function are discussed, as are the problems of parameterization and suboptimization. A solution procedure for the model is suggested, but an efficient algorithm is not available for solving the model. Some areas for future research are also suggested.

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| 14 KEY WORDS | LINK A | | LINK B | | LINK C | |
|---|---|---|---|---|---|---|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| Benefit Analysis | | | | | | |
| Development Program | | | | | | |
| Integer 0-1 Decision Variables | | | | | | |
| Nonlinear Integer Programming | | | | | | |
| Resource Allocation | | | | | | |
| Suboptimization | | | | | | |
| Transportation Network | | | | | | |
| Transportation System | | | | | | |
| Underdeveloped Country | | | | | | |